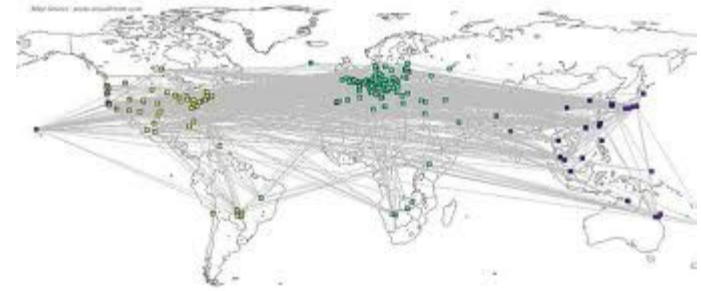


CSCD 433

Network Programming

Fall 2016



Lecture 5

Physical Layer Continued

Topics

- Definitions
- Analog Transmission of Digital Data
- Digital Transmission of Analog Data
- Multiplexing

Different Types of Channels

- Intended use of communication channel dictates whether Analog, Digital, Modulated or Unmodulated transmission is needed
 - Radio and Cellular communications require modulating signal to fit within specific frequency ranges
 - Use of telephone lines for digital data transmission requires different type of modulation to transmit digital over traditionally analog lines
- Look at analog signals over digital and digital signals over analog

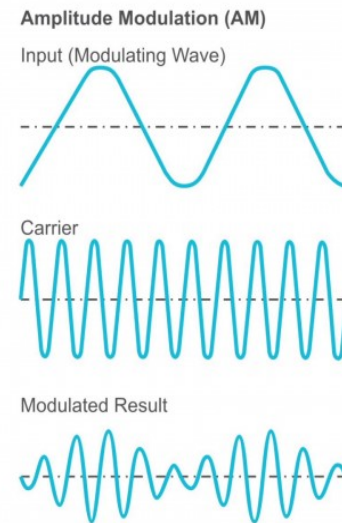
Definitions



- First, need to define terms
 - Carrier Wave
 - Modulation
 - Baseband and Bandpass Channels

Carrier Wave

- Carrier wave is pure wave of constant frequency, like a sine wave ... By itself it doesn't carry much information
- To include speech or data information, another wave needs to be imposed, called an **input signal**, on top of the carrier wave



- Imposing an input signal onto carrier wave is called modulation.

- Modulation changes shape of carrier wave to somehow encode the speech or data we need to encode

Modulation

Modulation is modification of carrier wave's fundamental characteristics in order to encode information

There are three basic ways to modulate an analog carrier wave:

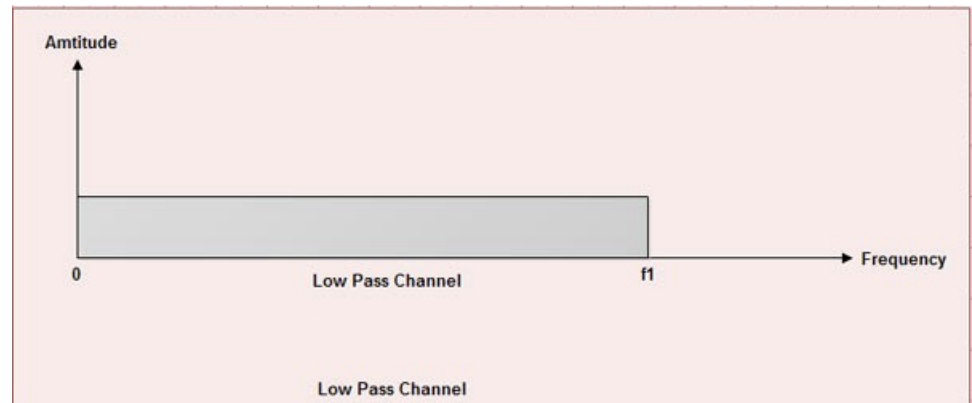
- Amplitude Modulation
- Frequency Modulation
- Phase Modulation

Baseband Channels

- Baseband transmission is transmission of encoded signal using its own baseband frequencies; i.e. without any shift to higher frequency ranges,
 - Without a shift in range of signal frequencies
 - Signal frequencies are within band of frequencies from near 0 hertz up to a higher cut-off frequency or maximum bandwidth

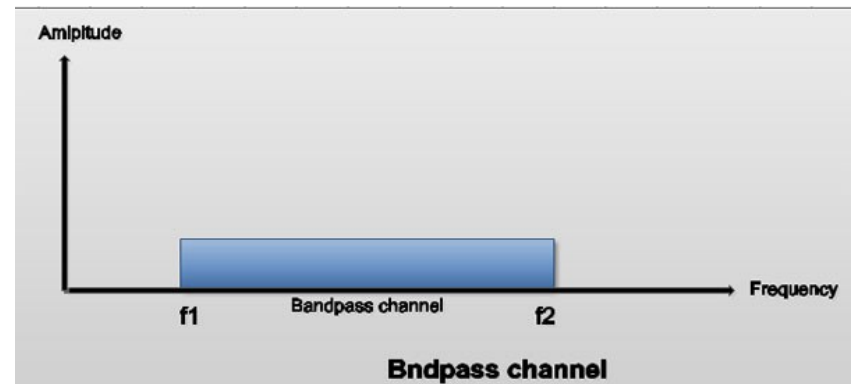
Examples:

Serial cables and local area networks utilize baseband transmission



Passband Channel

- A passband channel has range of frequencies or wavelengths that can pass through a filter
- In passband transmission, modulation methods are used so that only a limited frequency range is used
- Typically, range does not include 0 or low frequencies
- Example: Utilized in wireless communication and in radio transmission



Analog Transmission of Digital Data

- Traditional analog transmission has been Telephone service
 - **POTS – Plain Old Telephone Service**
 - Have an existing system of wired communication
 - Sending digital data over telephone service –
How do we do it?
 - **Use a modem !!!**
 - Modems use carrier waves to send information
 - Known as modulation

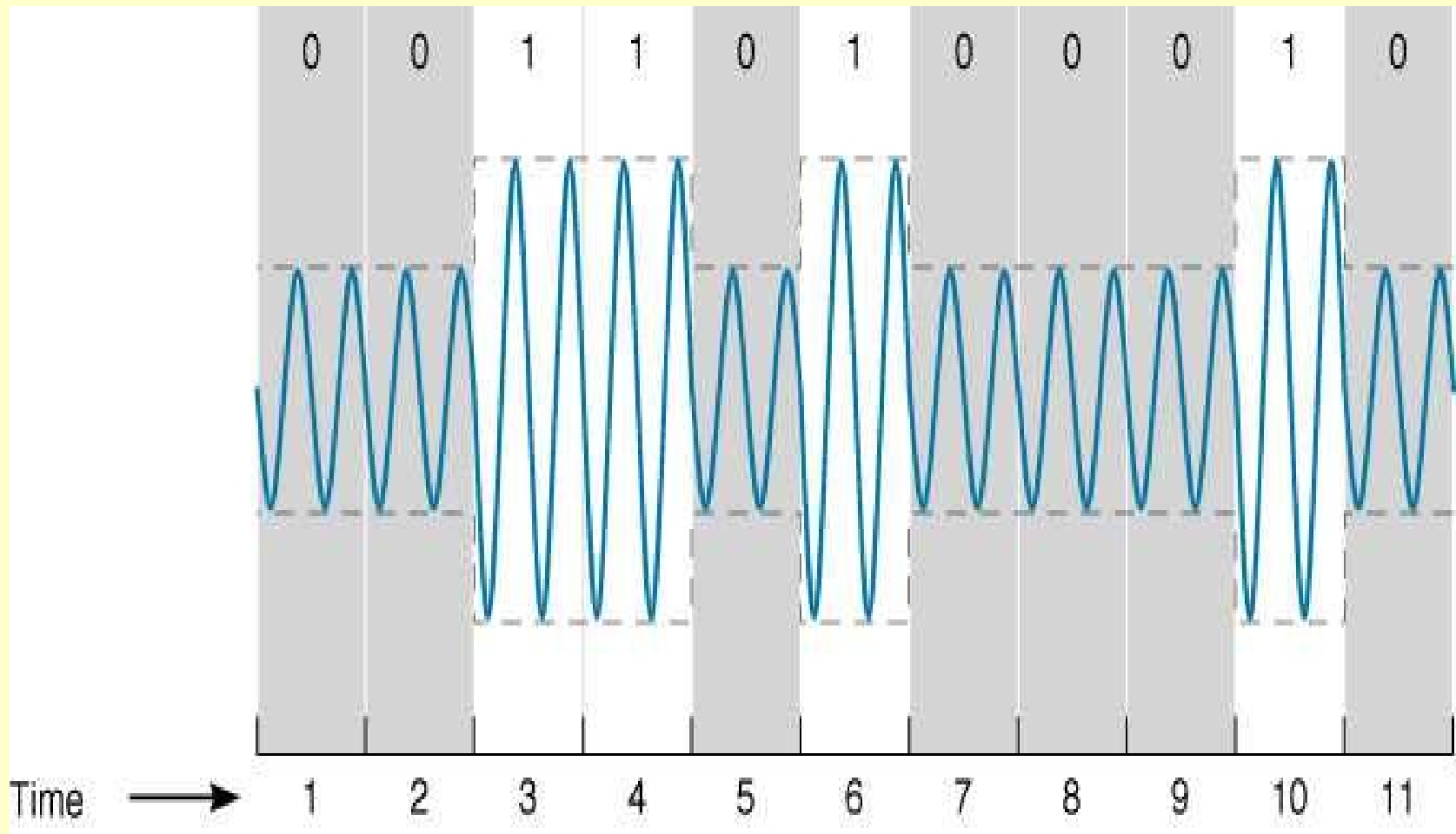
Amplitude Modulation

Amplitude Modulation (AM)

Amplitude Shift Keying (ASK), means changing height of wave to encode data

- AM dial on radio uses amplitude modulation to encode analog information
- Next Slide shows simple case of amplitude modulation in which one bit is encoded for each carrier wave change.
 - A high amplitude means a bit value of 1
 - Zero amplitude means a bit value of 0

Amplitude Modulation

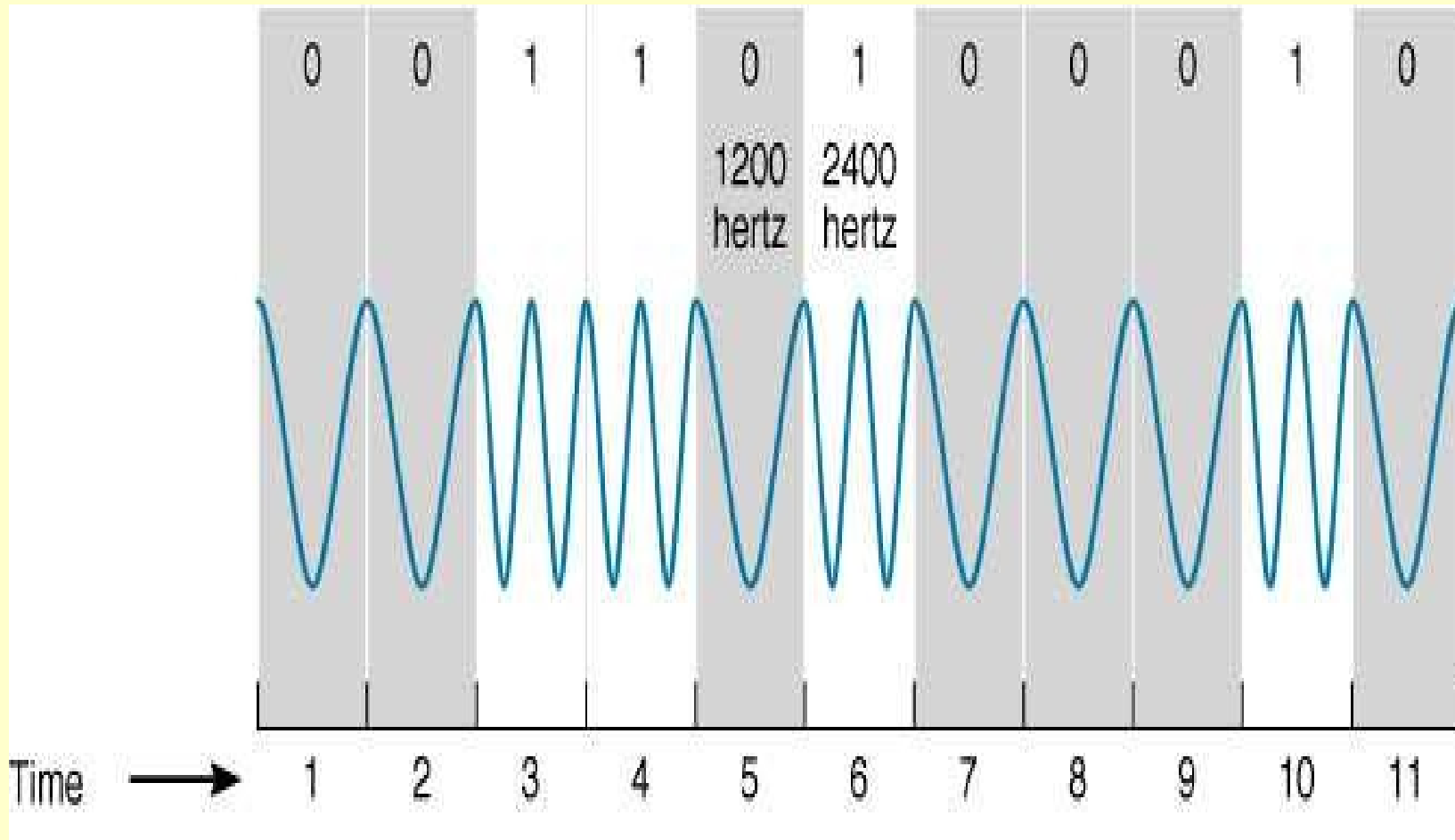


Frequency Modulation

Frequency Modulation (FM)

- Frequency Shift Keying (FSK), change frequency of carrier wave to encode data
 - FM radio uses frequency modulation to encode analog information
- Next slide shows frequency modulation in which one bit is encoded for each carrier wave change
 - Changing carrier wave to a higher frequency encodes a bit value of 1
 - No change in carrier wave frequency means a bit value of 0

Frequency Modulation



Phase Modulation

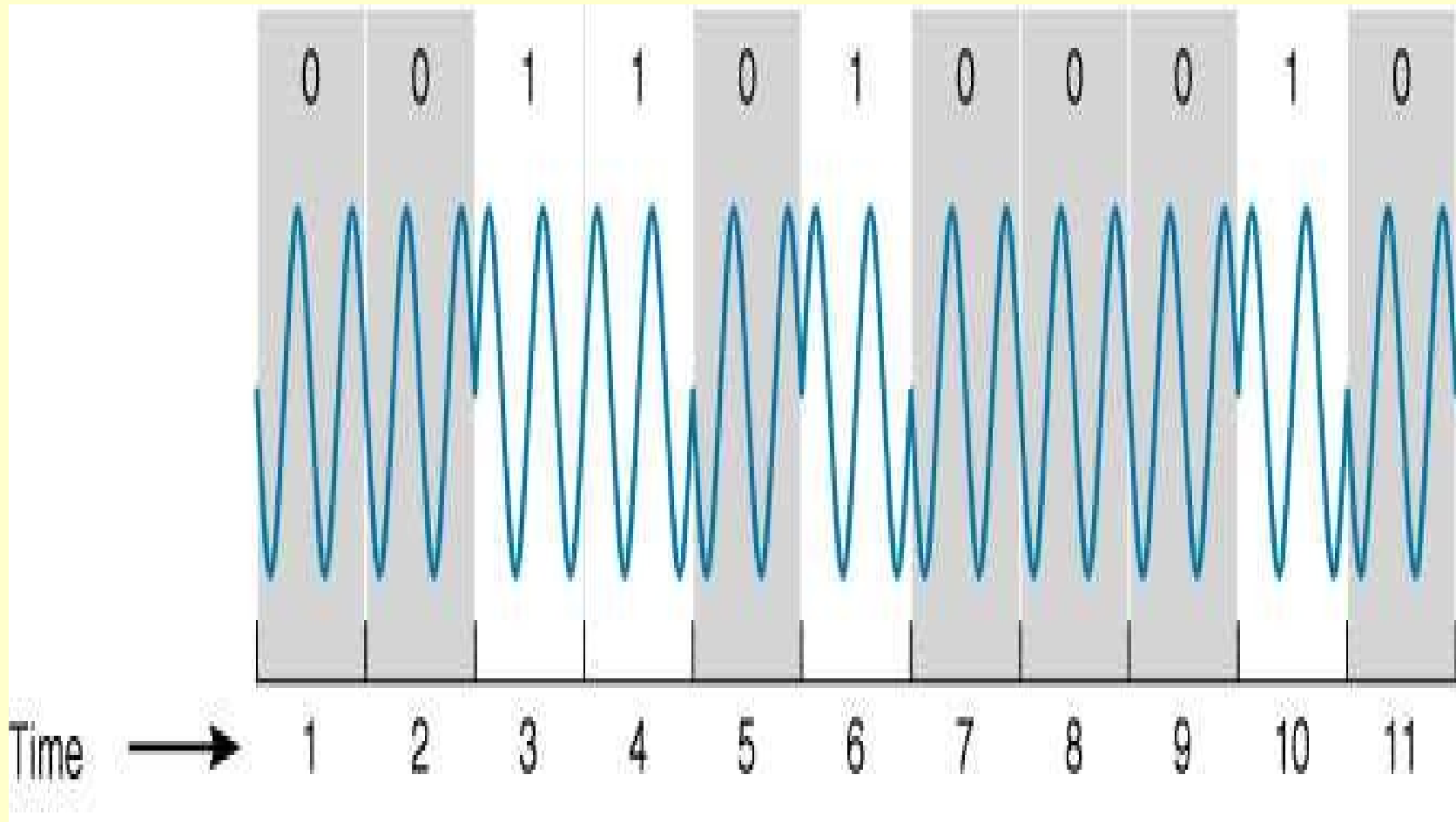
Phase refers to point in each wave cycle at which the wave begins

Phase Modulation (PM)

Phase Shift Keying (PSK) means changing phase of carrier wave to encode data

- **Next slide** shows phase modulation in which one bit is encoded for each carrier wave change
 - Changing the carrier wave's phase by 180° corresponds to a bit value of 1
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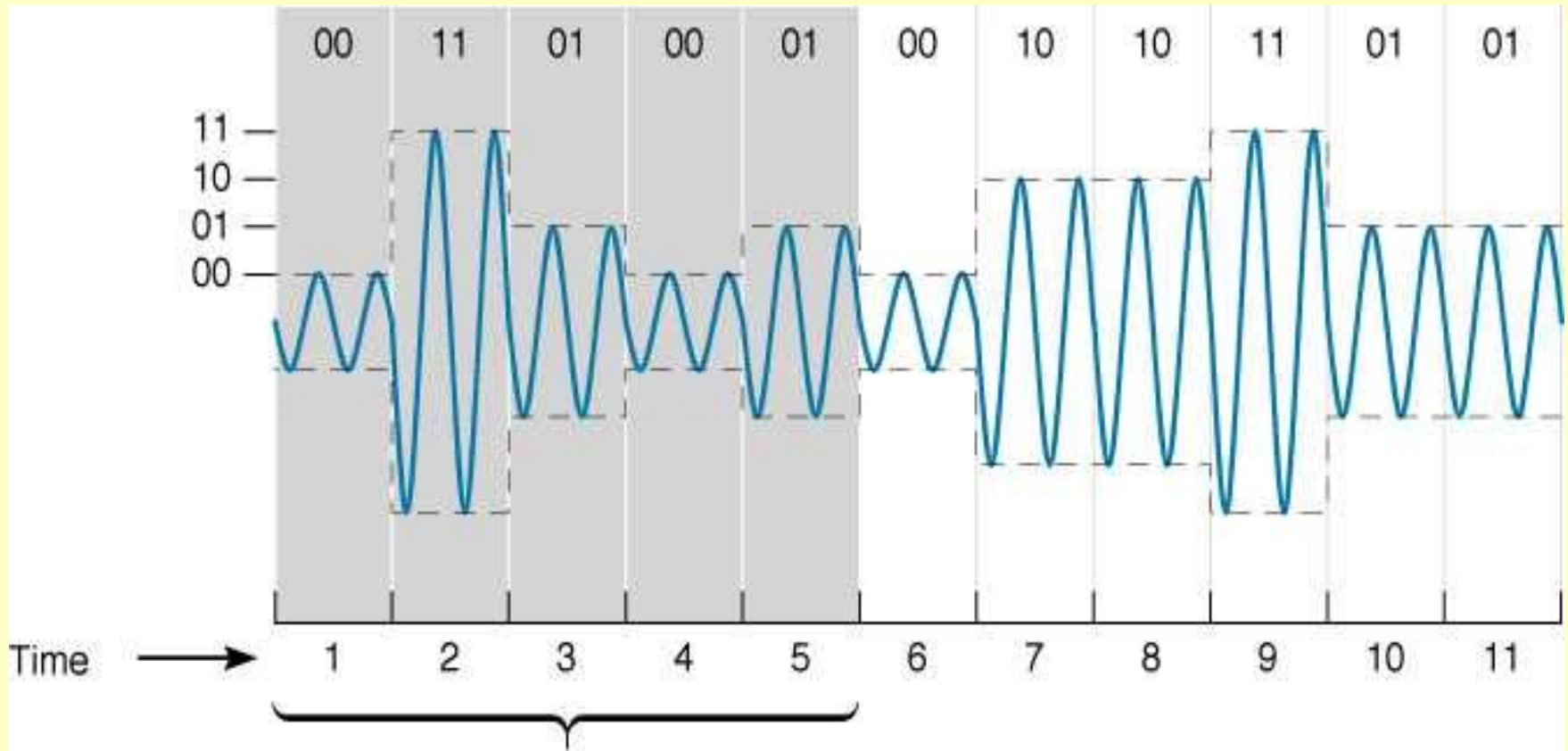
Phase Modulation



Sending Multiple Bits Simultaneously

- Each modification of carrier wave to encode information is called a **symbol**
 - By using more complicated information coding system, possible to encode more than 1 bit/symbol
- Next slide amplitude modulation using 4 amplitude levels, corresponding to 2 bits/symbol
- Increasing possible number of symbols from 4 to 8 corresponds with encoding 3 bits/symbol, 16 levels to 4 bits, and so on
- Likewise, multiple bits per symbol might be encoded using phase modulation, say using phase shifts of 0° , 90° , 180° , and 270°

Two-bit Amplitude Modulation



This data took 10 time steps
with 1-bit amplitude modulation.

Quadrature Amplitude Modulation (QAM)

- **QAM** is family of encoding schemes widely used for encoding multiple bits per symbol that combine Amplitude and Phase Modulation
- 16QAM uses 8 different phase shifts and 2 different amplitude levels
 - Since 16 possible symbols, each symbol encodes 4 bits
- QAM and related techniques are commonly used for voice modems

Bit Rate vs. Baud Rate (Symbol Rate)

Bit rate (or **data rate**) is number of bits transmitted per second

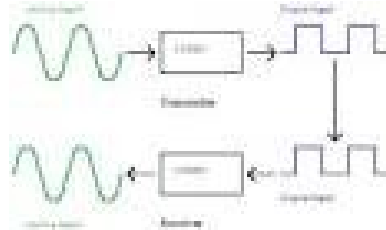
Baud rate (same as **symbol rate**) refers to number of symbols transmitted per second

Since multiple bits can be encoded per symbol, the two terms are not the same !!!!

General formula:

Data Rate (bits/second)=

Symbol Rate (symbols/sec.) x No. of bits/symbol



Digital Transmission of Analog Data

Analog Signal Over Digital System

- First you Digitize it
 - Obtain sample values
- Second you Quantize it
 - Decide how many bits needed to represent a sample



Of interest to us, is how many samples and how many bits to represent the samples ...

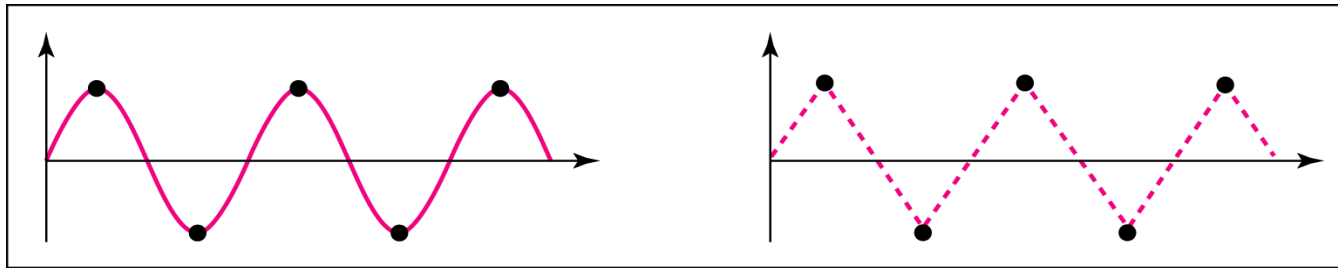
Sampling Theory for Signals

Nyquist sampling theorem

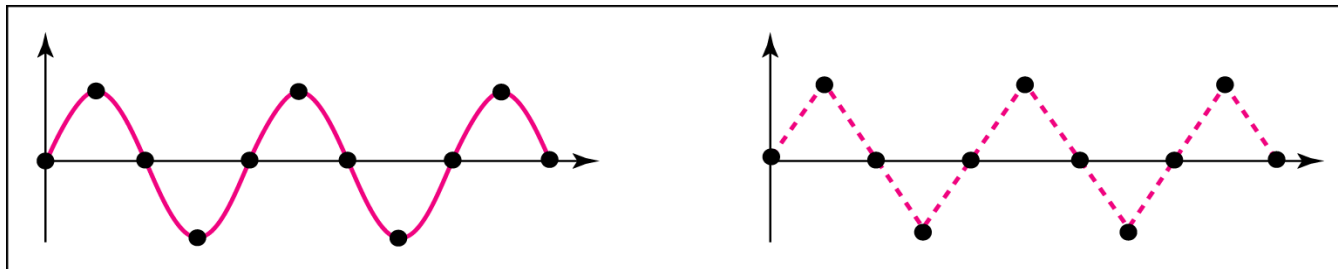
- Turns out there is another theorem for sampling an analog signal
- It is two times the signals maximum frequency
- If fewer samples are taken can't reconstruct signal very well

$$f_s \geq 2 \times f_{\max}$$

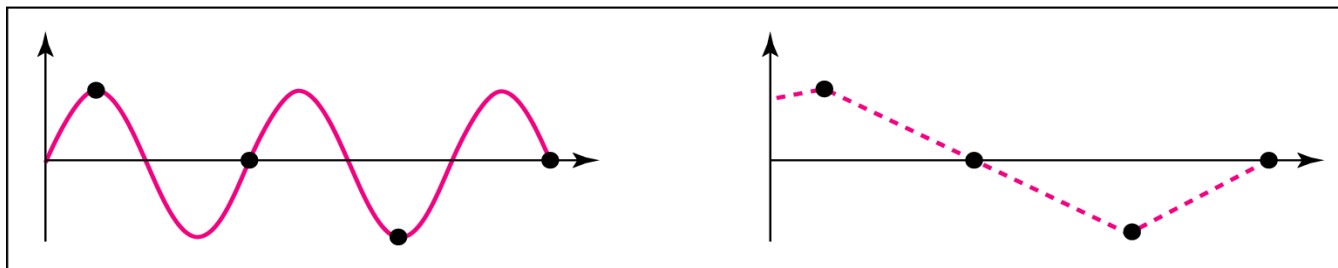
Figure 4.24 *Recovery of a sampled sine wave for different sampling rates*



a. Nyquist rate sampling: $f_s = 2 f$



b. Oversampling: $f_s = 4 f$



c. Undersampling: $f_s = f$

Enough Bits to Represent Signal

- Recall the Nyquist Theorem

$$\text{Bit Rate} = 2 \times \text{Bandwidth} \times \log_2 L$$

L = number of signal levels

- Signal samples can still be large mathematical numbers
- **Quantization** makes range of signal discrete, so that quantized signal takes on only discrete, usually finite, set of values
- This is usually good enough for human ear to pick up sound

Enough Bits to Represent Signal

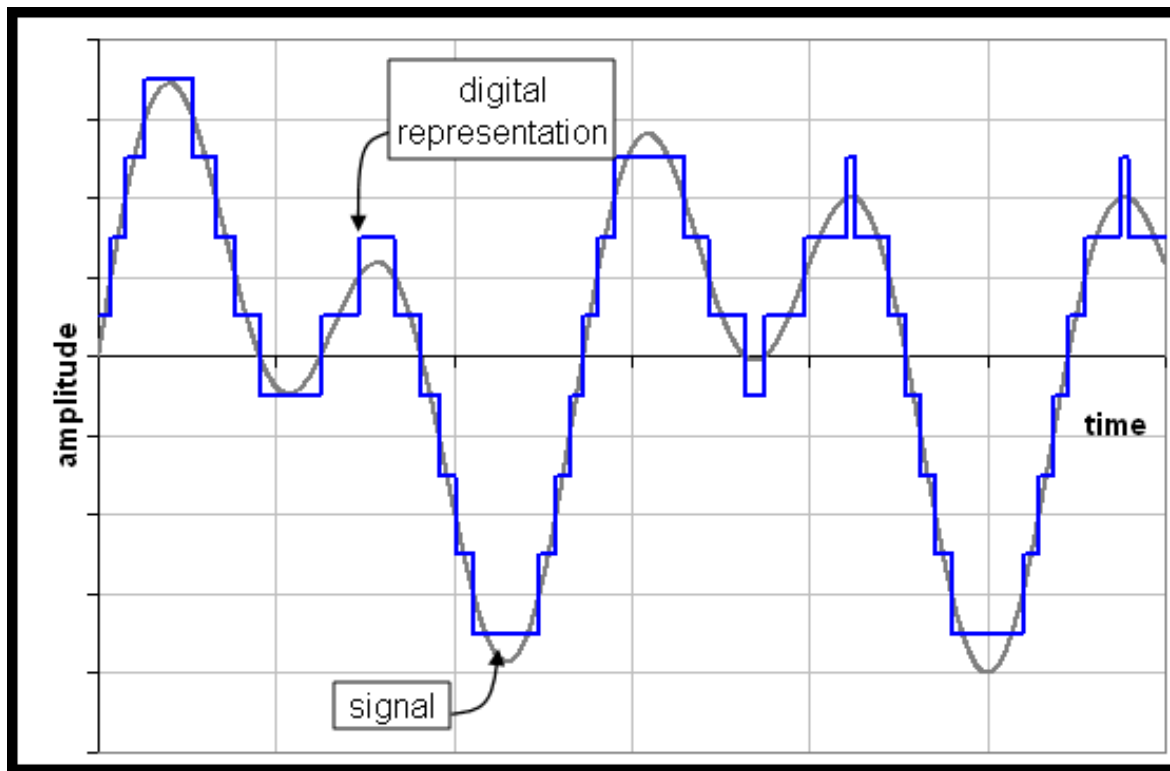
- With L levels need $N = \log_2 L$ bits to represent the different levels, or conversely, with N bits we can represent $L = 2^N$ levels
- Simplest type of quantizers are called zero memory quantizers in which quantizing a sample is independent of other samples.
- Signal amplitude is represented using some finite number of bits independent of sample time and independent of values of neighboring samples

Quantization Defined

- **Quantization**, in mathematics and digital signal processing, is
 - Process of mapping a large set of input values to a (countable) smaller set.
 - Rounding and truncation are typical examples of quantization processes.

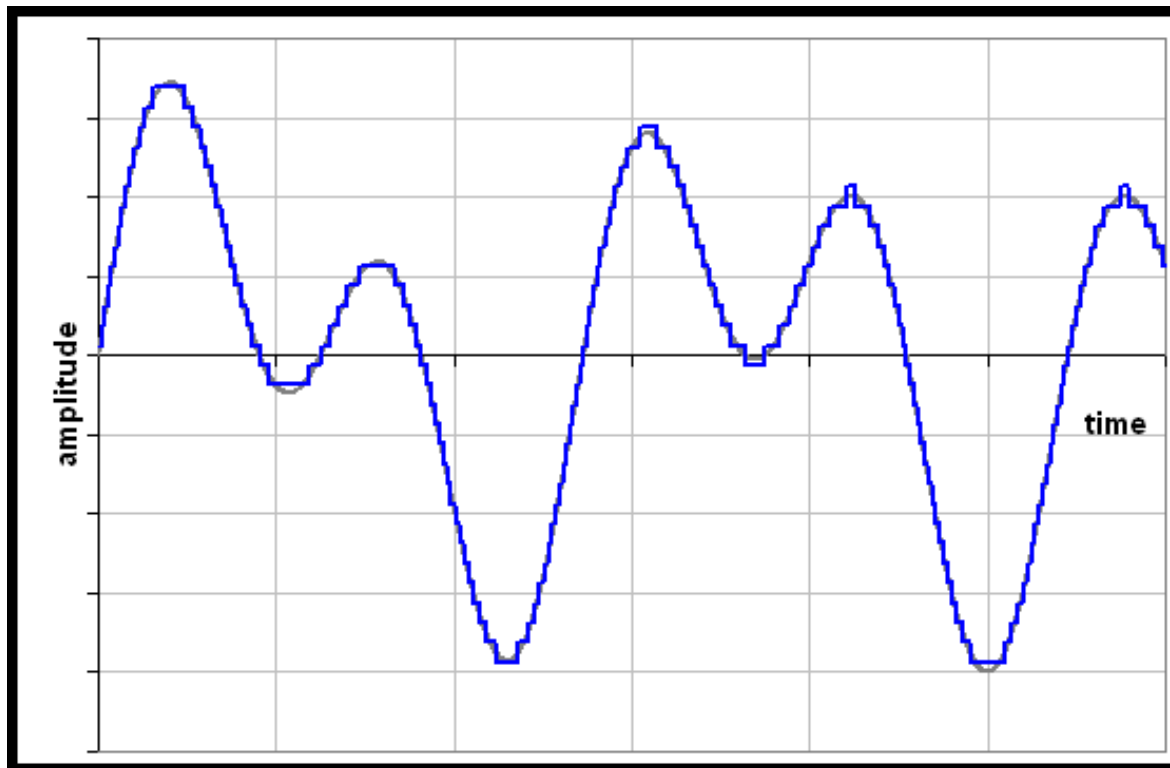
Quantized View of Signal – 3 bit

For a real signal digitising this with 3 bit values gives a very poor representation of the signal.



Quantized View of Signal – 5 bit

Increase the sample size to 5 bit and the digital representation is much closer to the original signal.

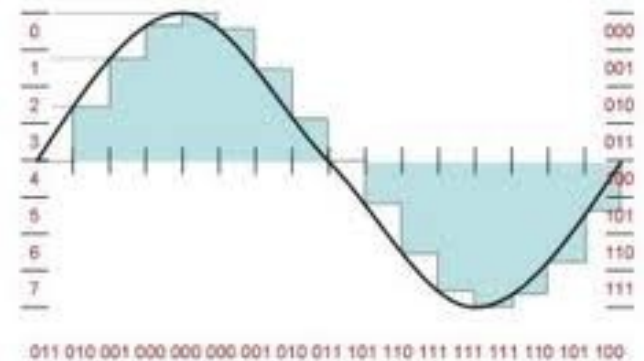


Techniques for Quantifying Analog Signals

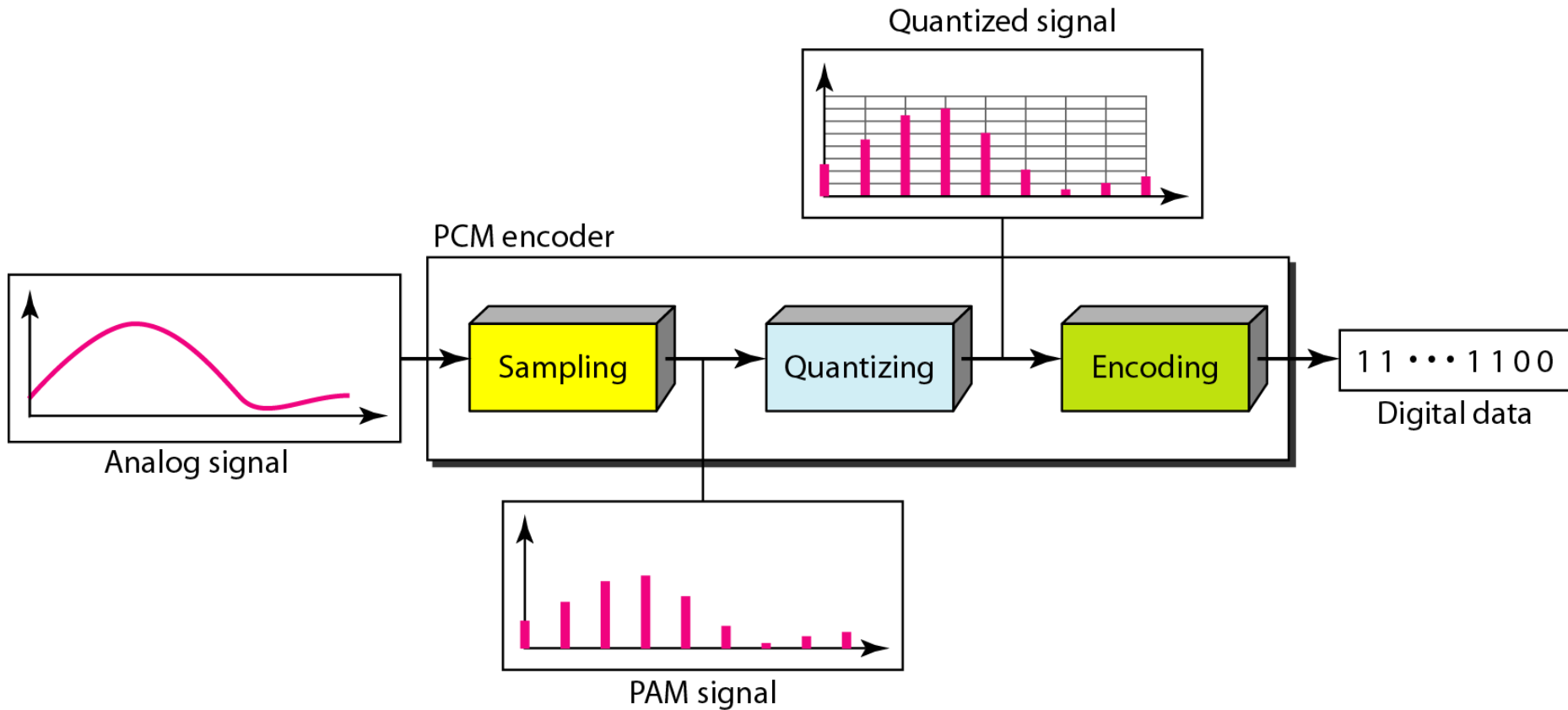
- **Pulse-code modulation (PCM)** is a method used to digitally represent sampled analog signals
 - It is standard form of digital audio in computers, Compact Discs, telephony and other digital audio applications
 - In PCM stream, amplitude of analog signal is sampled regularly at uniform intervals, and each sample is quantized to nearest value within a range of digital steps

Pulse Code Modulation

- Analog signal amplitude is sampled (measured) at regular time intervals.
- Want to take several times maximum frequency of the analog waveform in cycles per second or hertz
- Amplitude of analog signal at each sampling is rounded off to nearest of several specific, predetermined levels
- **Quantization of the signal**



Components of PCM Encoder



Components of a PCM Decoder

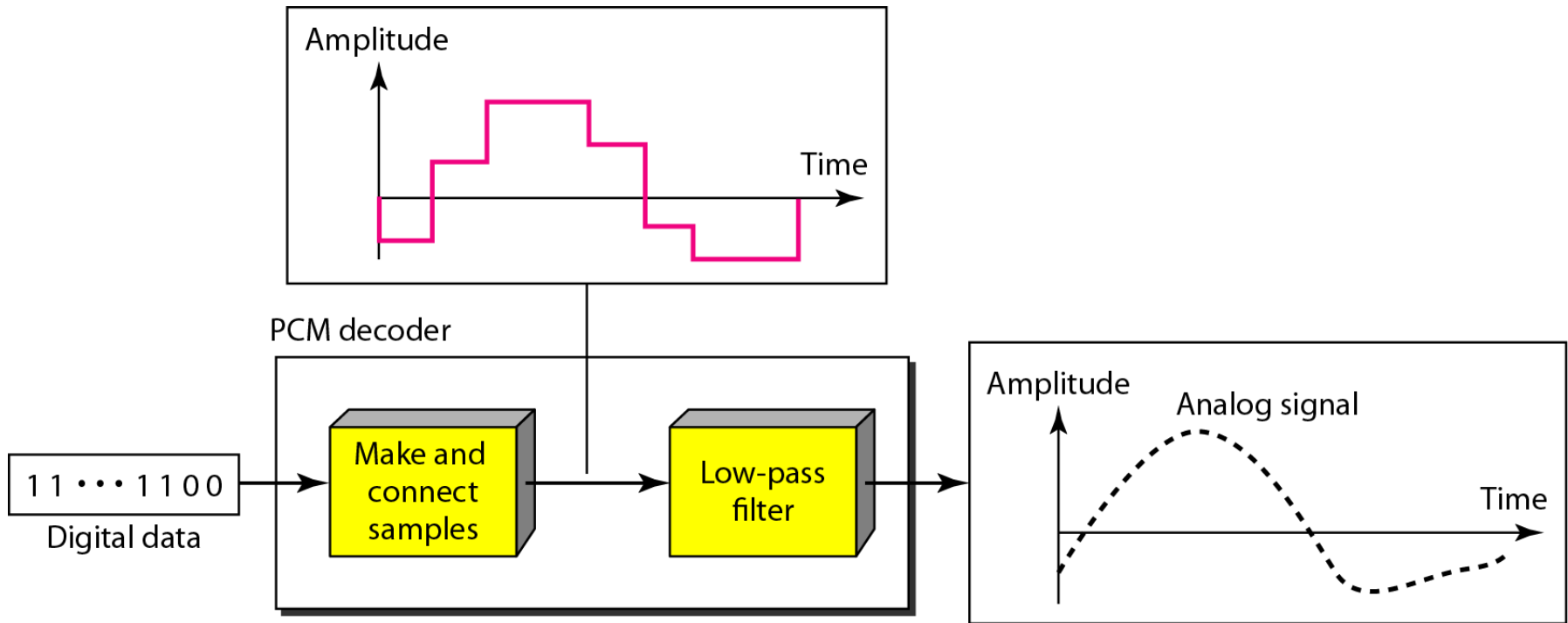
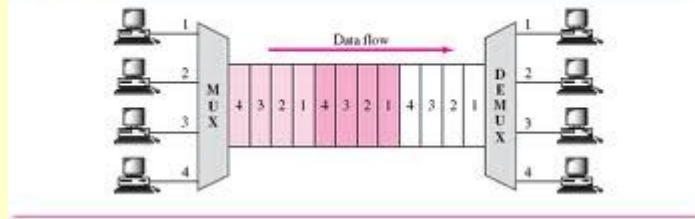


Figure 6.12 TDM



Multiplexing

Multiplexing

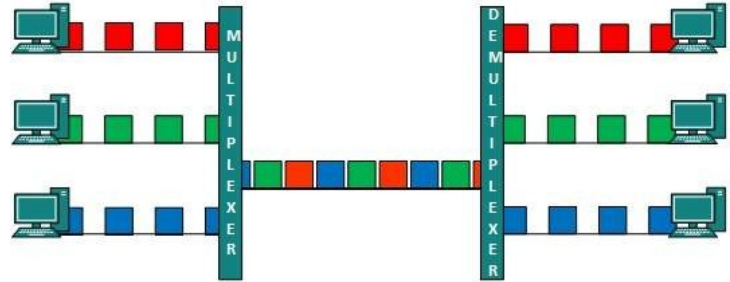
What is multiplexing?

Multiplexing

Combining several signals onto one line.

Demultiplexing

Taking a multiplexed signal and recovering its original components



Multiplexing

- **Frequency division multiplexing (FDM):** Use different frequency ranges for different signals
- **Wave division multiplexing (WDM):** Same as FDM, but with optical signals
- **Time division multiplexing (TDM):** Each signal is allocated to a periodic time slot
- **Code division multiplexing (CDM):** is mathematical approach used in **cell phone and wireless**

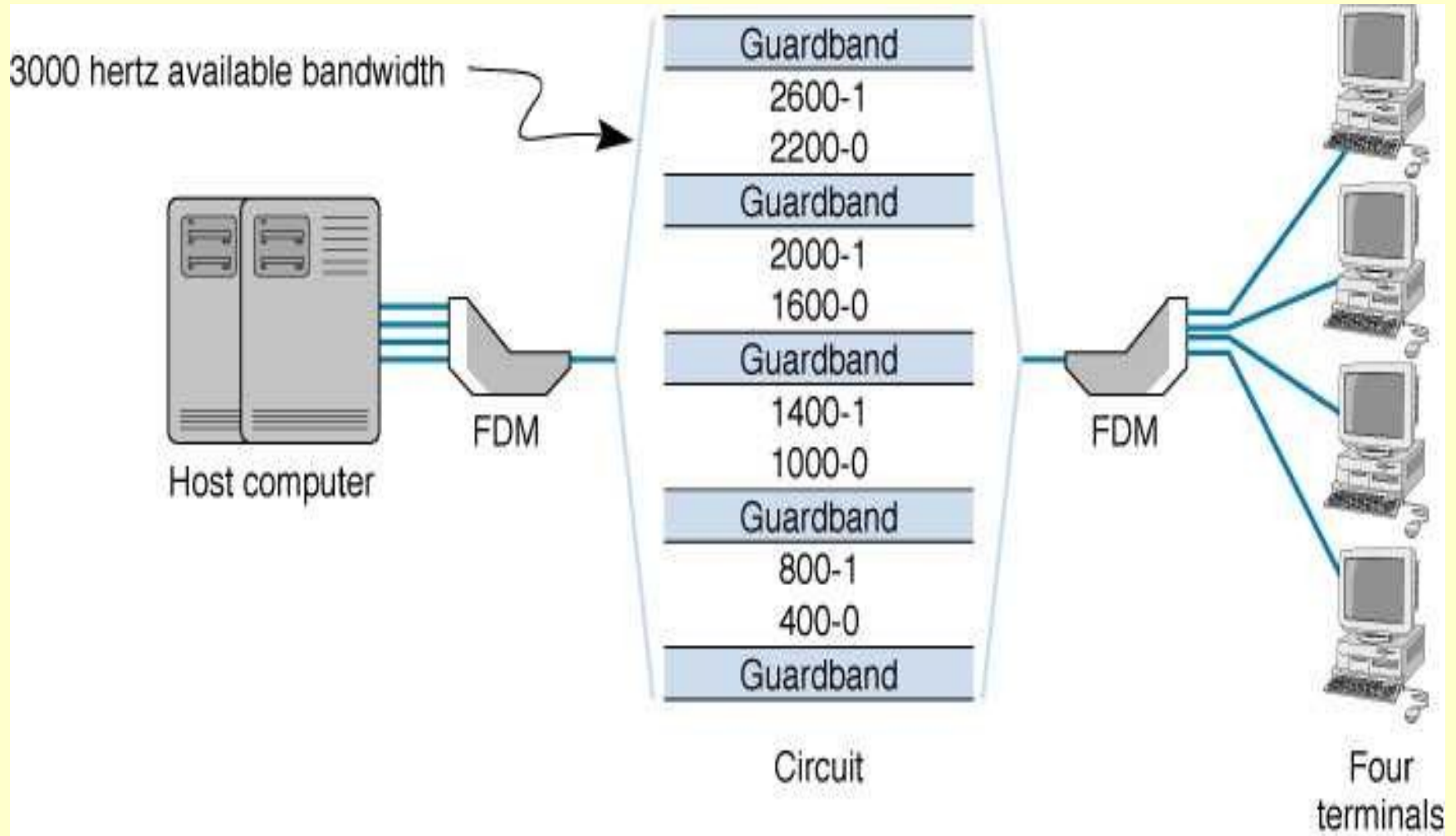
Frequency Division Multiplexing (FDM)

FDM works by making number of smaller channels from larger frequency band

FDM is sometimes referred to as dividing the circuit “horizontally”

- In order to prevent interference between channels, unused frequency bands called guardbands separate channels
- Because of guardbands, there is some wasted capacity on an FDM circuit

Frequency Division Multiplexing



Frequency Division Multiplexing

Example:

Suppose have three phone signals want to combine onto one line with higher bandwidth.

Allocate 4 KHz of bandwidth to each signal, which includes a “guard band” of unused frequency range so signals don’t overlap.

Each signal originally uses range 0.3 – 3.3 KHz.

Transform each signal to a different frequency range. Just learned this ...

Signal 1: 20 – 24 KHz channel

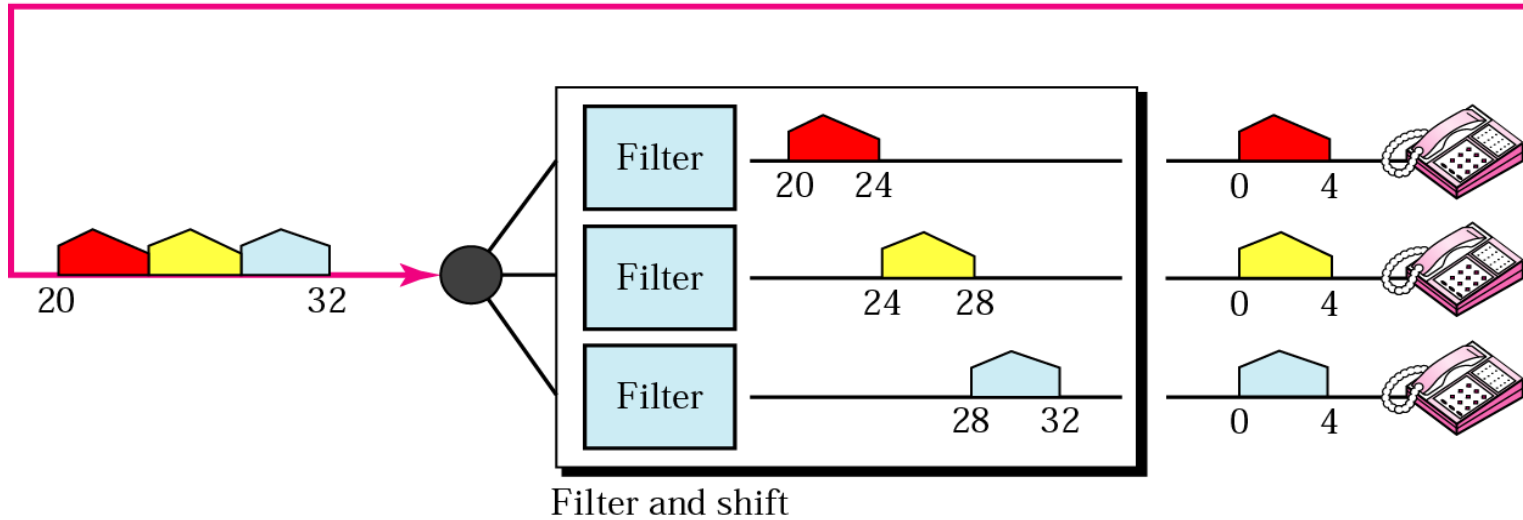
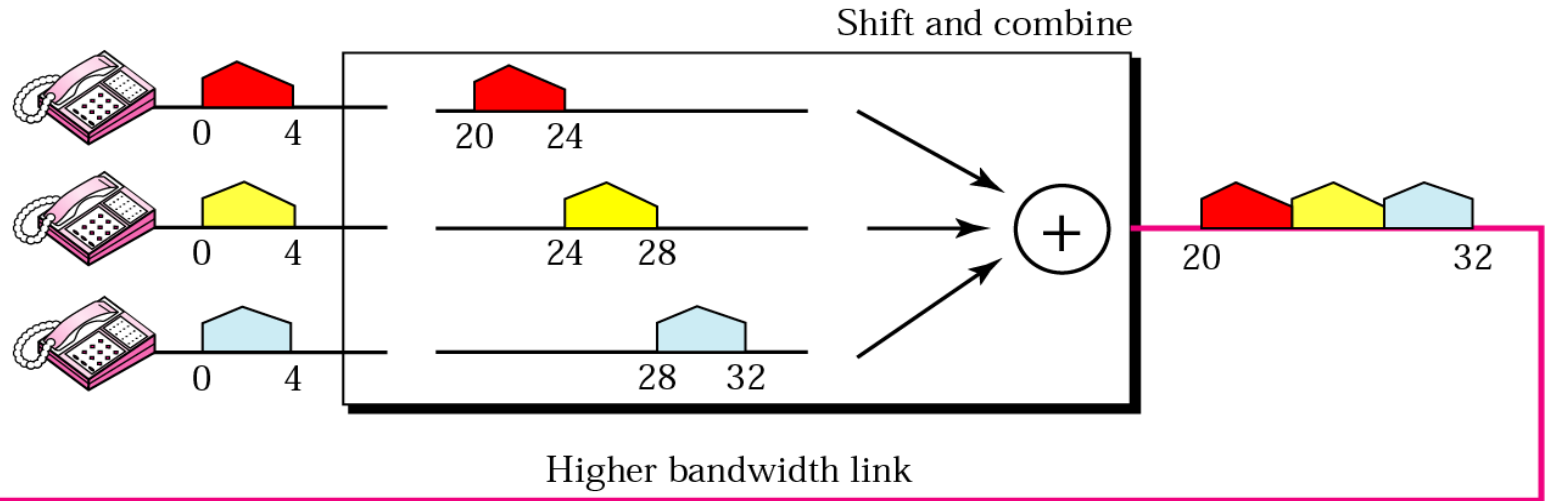
- Use 20.5 KHz to 23.5 KHz, with 0.5 KHz of guard band on each end.

Signal 2: 24 – 28 KHz

Signal 3: 28 – 32 KHz

At receiver, filters are used to isolate each channel, and then the frequency is transformed back to its original range.

FDM



FDM applications

High capacity phone lines

AM radio: 530 KHz to 1700 KHz, 10 KHz bandwidth per station

FM radio: 88 MHz to 108 MHz, 200 KHz bandwidth per station

TV broadcasts: 6 MHz bandwidth per TV channel

First generation cell phones: each user got two 30 KHz channels (sending, receiving).

Wave Division Multiplexing

Essentially same as FDM, except signals are optical and prisms are used to combine/split signals instead of electrical components.

Used to combine signals of different frequencies (i.e. colours) onto one fibre-optic cable

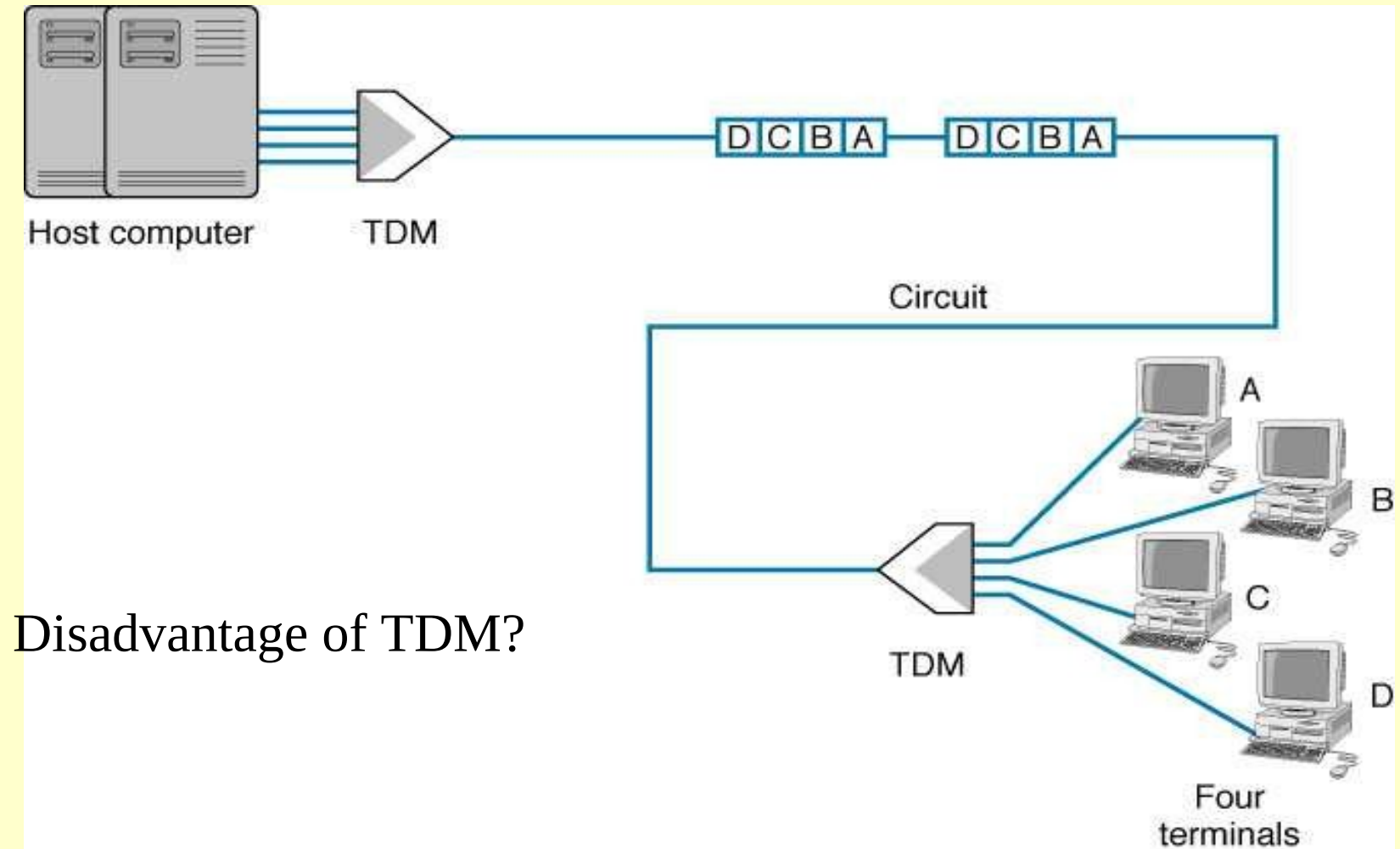
Time Division Multiplexing (TDM)

TDM allows multiple channels by allowing channels to send data by “taking turns”

TDM is sometimes referred to as dividing circuit “vertically”

- [Next slide](#) shows 4 terminals sharing a circuit, with each terminal sending one character at a time
- With TDM, time on circuit is shared equally each channel gets a time slot, whether or not it has any data to send
- TDM is more efficient than FDM, since TDM doesn't use guardbands, so entire capacity can be divided between channels

Time Division Multiplexing



Disadvantage of TDM?

TDM applications

Digital Service lines: DS-n

Implemented as telephone lines: T-n

Service	Phone line	Data rate	# of voice channels
(DS-0)	standard phone line	64 Kb/s	1
DS-1	T-1	1.544 Mb/s	24
DS-2	T-2	6.312 Mb/s	96
DS-3	T-3	44.736 Mb/s	672
DS-4	T-4	274.176 Mb/s	4032

Code Division Multiplexing (CDM)

CDM used in parts of cellular telephone system and for some satellite communication

CDM relies on an interesting mathematical idea values from **orthogonal vector** spaces can be combined and separated without interference

Each **sender** is assigned a unique binary code C_i that is known as a **chip sequence**

chip sequences are selected to be orthogonal vectors

- Means **dot product** of any two chip sequences is zero

Code Division Multiplexing (CDM)

At any point in time, each sender has value to transmit, V_i

The senders each multiply $C_i \times V_i$ and transmit the results

The senders transmit at the same time and the values are added together

To extract value V_i , a receiver multiplies the sum by C_i

Example

Make example easy to understand, use chip sequence that's only **two bits** long and data values that are **four bits** long

think of the chip sequence as a vector

Next slide lists the values

Code Division Multiplexing

Sender	Chip Sequence	Data Value
A	1 0	1 0 1 0
B	1 1	0 1 1 0

Figure 11.15 Example values for use with code division multiplexing.

Code Division Multiplexing

The first step consists of converting the binary values into vectors that use **-1** to represent **0**:

$$C_1 = (1, -1) \quad V_1 = (1, -1, 1, -1) \quad C_2 = (1, 1) \quad V_2 = (-1, 1, 1, -1)$$

Multiplying $C_1 \times V_1$ and $C_2 \times V_2$ produces:

$$((1, -1), (-1, 1), (1, -1), (-1, 1)) \quad ((-1, -1), (1, 1), (1, 1), (-1, -1))$$

If we think of the resulting values as a combined signal to be transmitted at the same time

the resulting signal will be the sum of the two signals

	1	-1	-1	1	1	-1	-1	1
+	-1	-1	1	1	1	1	-1	-1
<hr/>								
	0	-2	0	2	2	0	-2	0

Code Division Multiplexing

A receiver treats the sequence as a vector

computes product of vector and the chip sequence

treats result as a sequence, and converts the result to binary by interpreting positive values as binary **1** and negative values as **0**

Thus, **Receiver 1** computes:

$$C_1 \rightarrow (1, -1) \cdot \underline{((0, -2), (0, 2), (2, 0), (-2, 0))}$$

to get:

Received data

$$((0 + 2), (0 - 2), (2 + 0), (-2 + 0))$$

Interpreting the result as a sequence produces: **(2 -2 2 -2)**

which becomes the binary value: **(1 0 1 0)**

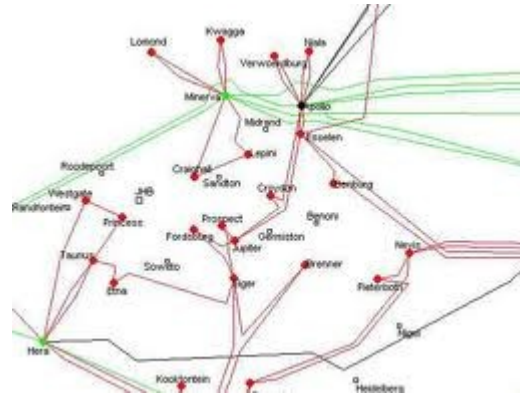
note that **1010** is the correct value of V_1

Receiver 2 will extract V_2 from the same transmission

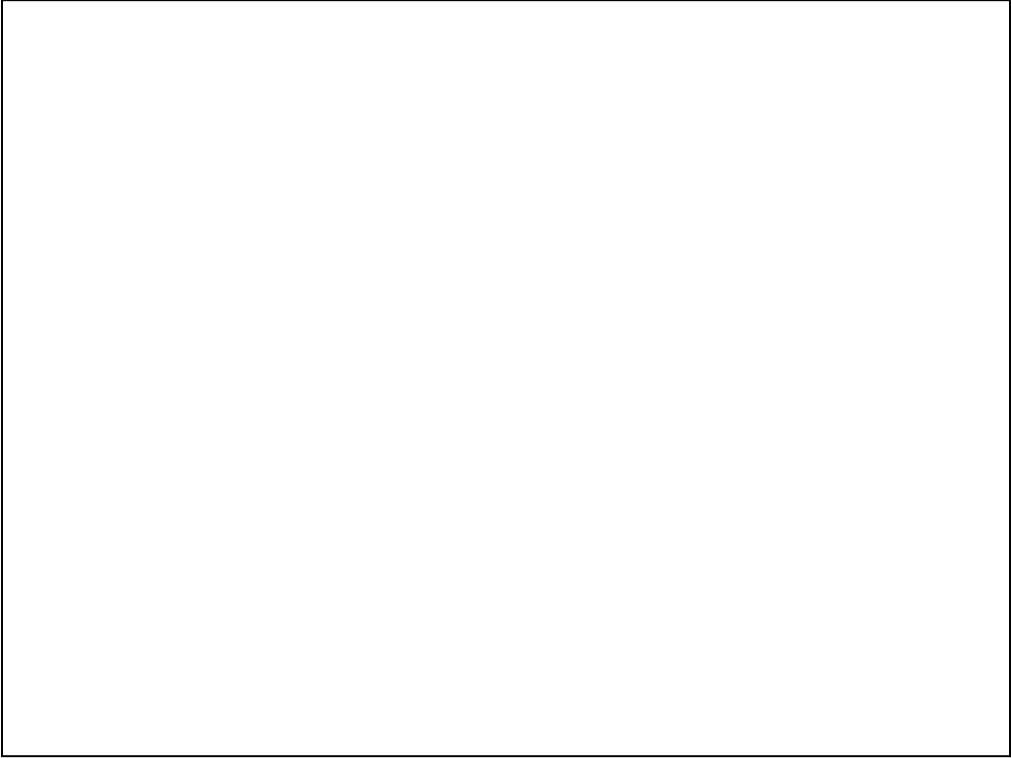
Code division multiple access (CDMA) is a channel access method utilized by various radio communication technologies.

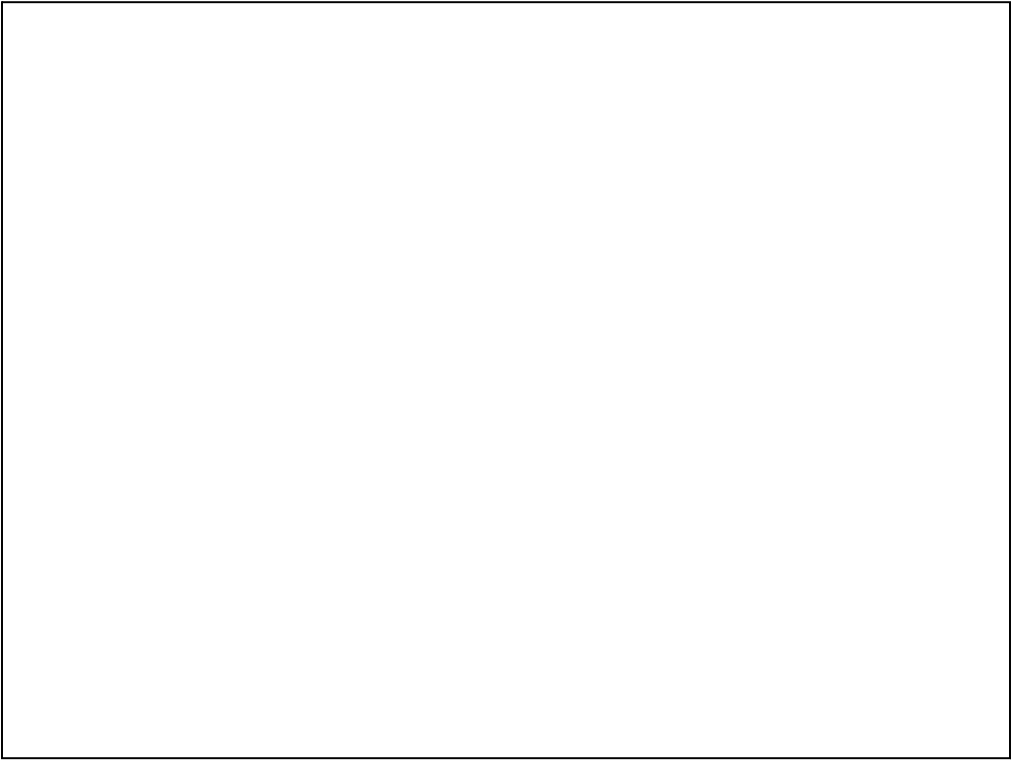
Summary

- Many types of encoding for sending data
- Other than LAN or switch connections, most communications require signal transforming
- Multiplexing allows sharing for efficient use of physical media
- Many interesting ways to make physical network communications more efficient



Keep working on HW 2





Different Types of Channels

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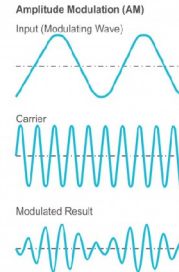
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Carrier Wave

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- To include speech or data information, another wave needs to be imposed, called an **input signal**, on top of the carrier wave



- Imposing an input signal onto carrier wave is called modulation.

- Modulation changes shape of carrier wave to somehow encode the speech or data we need to encode

What does it mean to modulate? What kinds are there? 5

Modulation

Modulation is modification of carrier wave's fundamental characteristics in order to encode information

There are three basic ways to modulate an analog carrier wave:

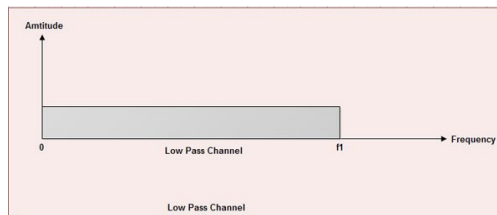
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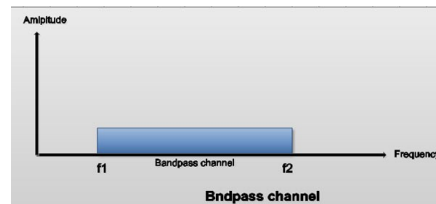
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- Example: Utilized in wireless communication and in radio transmission



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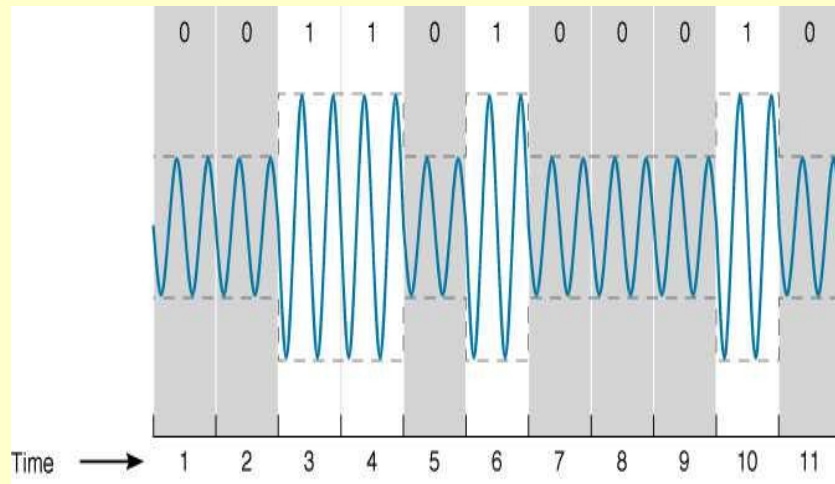
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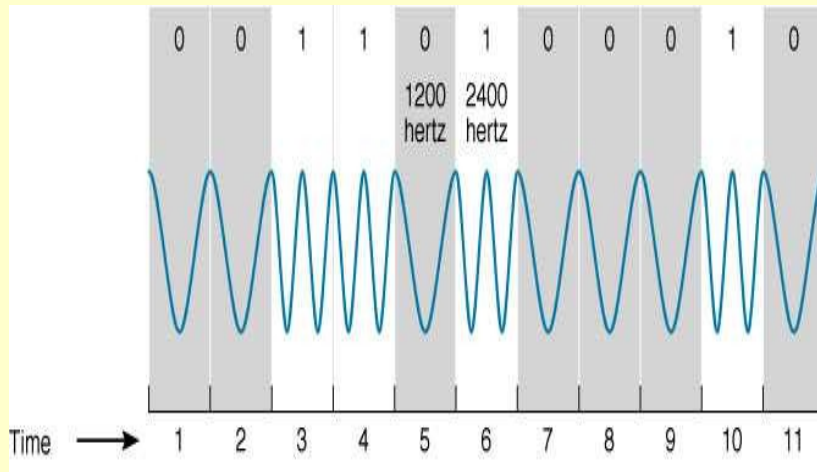


Frequency Modulation

Frequency Modulation (FM)

- Frequency Shift Keying (FSK), change frequency of carrier wave to encode data
 - FM radio uses frequency modulation to encode analog information
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Frequency Modulation



Phase Modulation

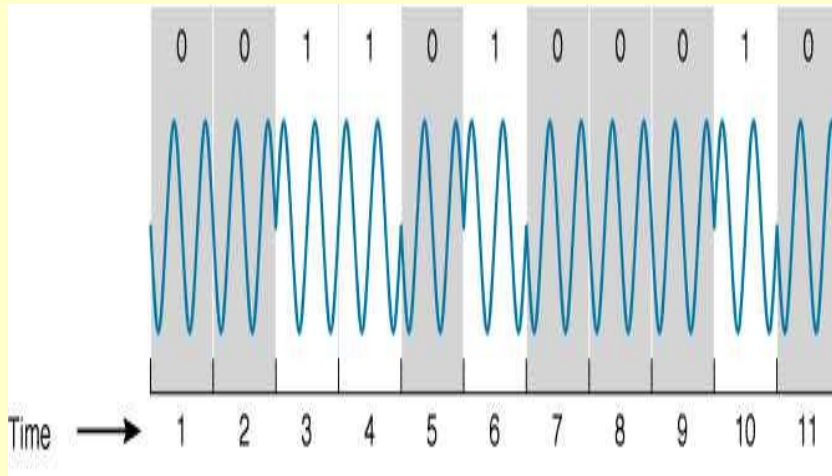
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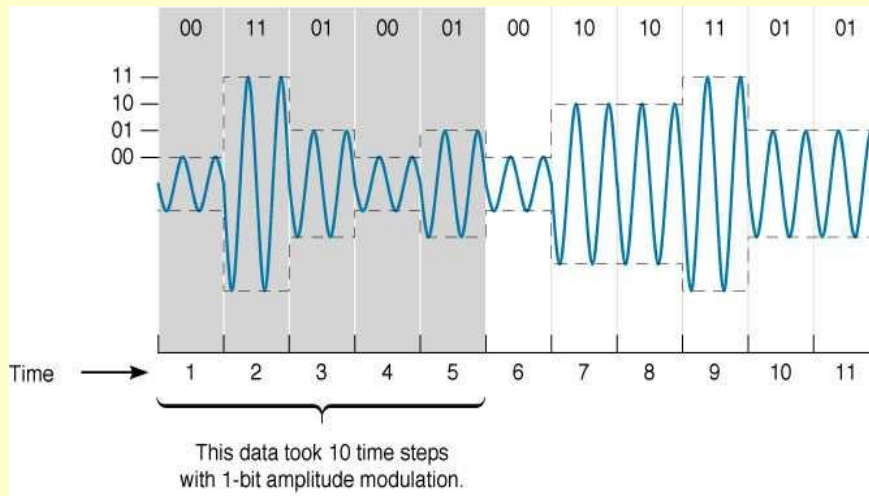
Phase Modulation



Sending Multiple Bits Simultaneously

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- 16QAM uses 8 different phase shifts and 2 different amplitude levels

Since 16 possible symbols, each symbol encodes 4 bits

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Bit Rate vs. Baud Rate (Symbol Rate)

Bit rate (or **data rate**) is number of bits transmitted per second

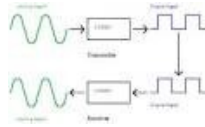
Baud rate (same as **symbol rate**) refers to number of symbols transmitted per second

Since multiple bits can be encoded per symbol, the two terms are not the same !!!!

General formula:

Data Rate (bits/second)=

Symbol Rate (symbols/sec.) x No. of bits/symbol



Digital Transmission of Analog Data

Analog Signal Over Digital System

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 - Decide how many bits needed to represent a sample



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Nyquist sampling theorem

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- It is two times the signals maximum frequency
- If fewer samples are taken can't reconstruct signal very well

$$f_s \geq 2 \times f_{\max}$$



Enough Bits to Represent Signal

- Recall the Nyquist Theorem

$$\text{Bit Rate} = 2 \times \text{Bandwidth} \times \log_2 L$$

L = number of signal levels

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Enough Bits to Represent Signal

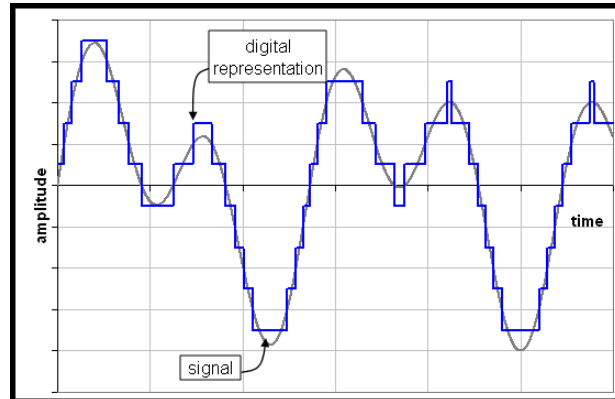
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 - Process of mapping a large set of input values to a (countable) smaller set.
 - Rounding and truncation are typical examples of quantization processes.

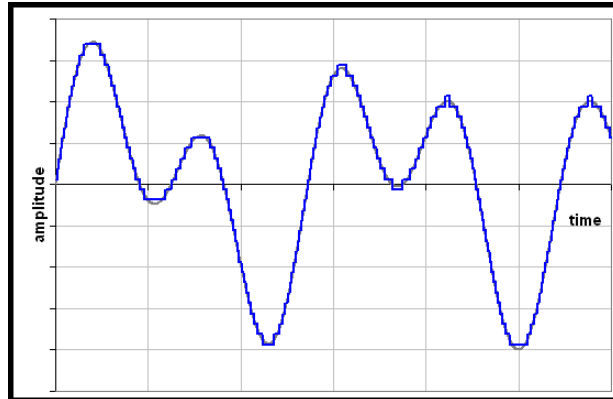
Quantized View of Signal – 3 bit

For a real signal digitising this with 3 bit values gives a very poor representation of the signal.



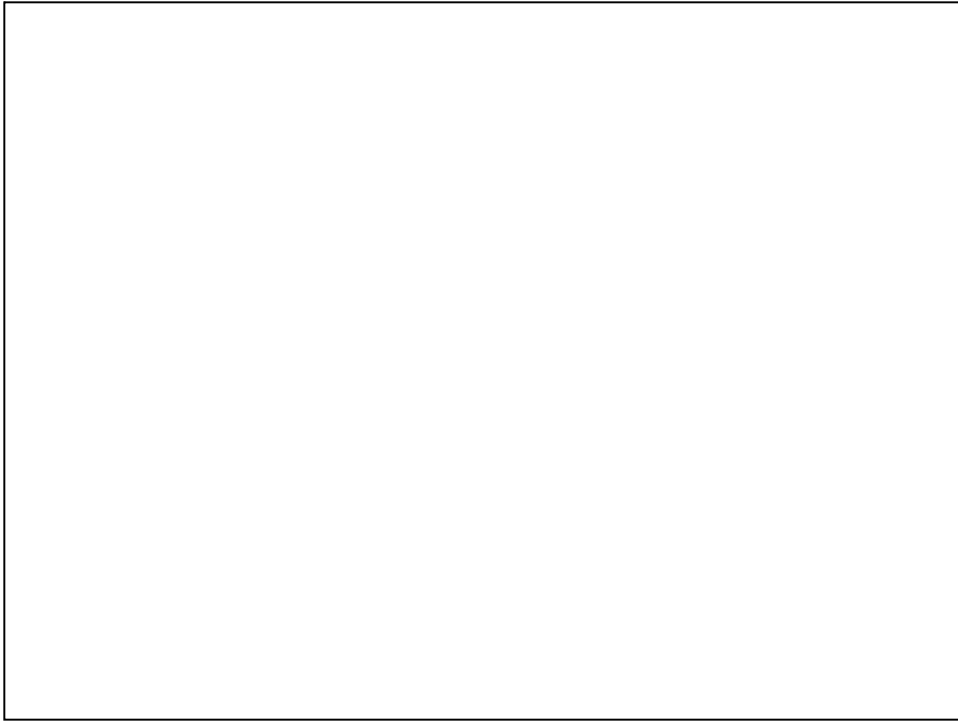
Quantized View of Signal – 5 bit

Increase the sample size to 5 bit and the digital representation is much closer to the original signal.

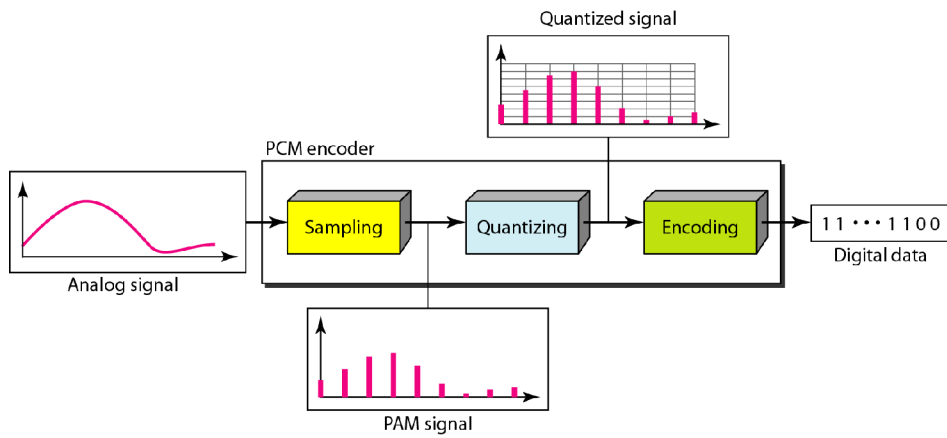


Techniques for Quantifying Analog Signals

- **Pulse-code modulation (PCM)** is a method used to digitally represent sampled analog signals
 - It is standard form of digital audio in computers, Compact Discs, telephony and other digital audio applications
 - In PCM stream, amplitude of analog signal is sampled regularly at uniform intervals, and each sample is quantized to nearest value within a range of digital steps

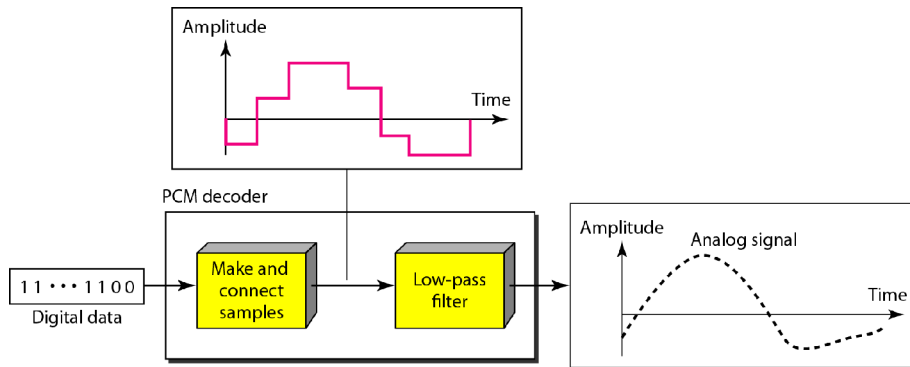


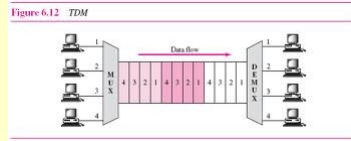
Components of PCM Encoder



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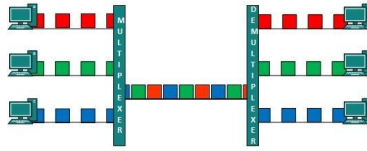
Components of a PCM Decoder





Multiplexing

Multiplexing



What is multiplexing?

Multiplexing

Combining several signals onto one line.

Demultiplexing

Taking a multiplexed signal and recovering its original components

Multiplexing

- **Frequency division multiplexing (FDM):** Use different frequency ranges for different signals
- **Wave division multiplexing (WDM):** Same as FDM, but with optical signals
- **Time division multiplexing (TDM):** Each signal is allocated to a periodic time slot
- **Code division multiplexing (CDM):** is mathematical approach used in **cell phone and wireless**

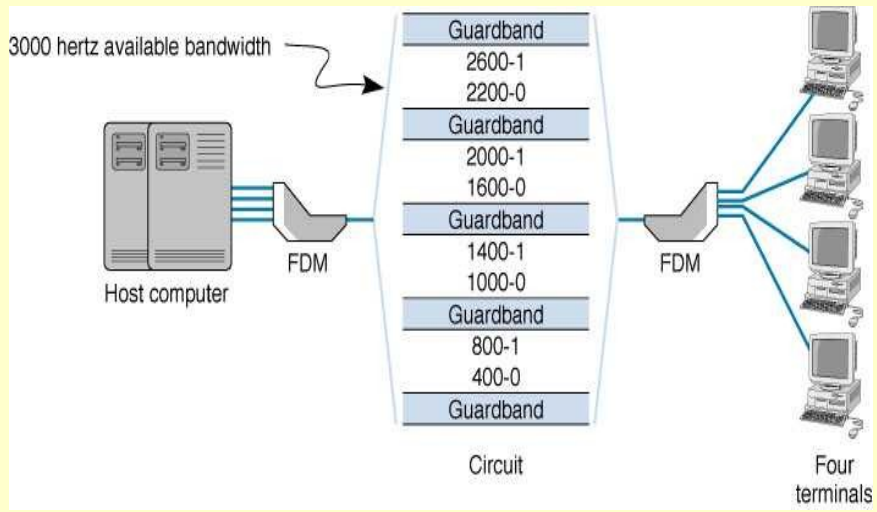
Frequency Division Multiplexing (FDM)

FDM works by making number of smaller channels from larger frequency band

FDM is sometimes referred to as dividing the circuit “horizontally”

- In order to prevent interference between channels, unused frequency bands called guardbands separate channels
- Because of guardbands, there is some wasted capacity on an FDM circuit

Frequency Division Multiplexing



Frequency Division Multiplexing

Example:

Suppose have three phone signals want to combine onto one line with higher bandwidth.

Allocate 4 KHz of bandwidth to each signal, which includes a "guard band" of unused frequency range so signals don't overlap.

Each signal originally uses range 0.3 – 3.3 KHz.

Transform each signal to a different frequency range. Just learned this ...

Signal 1: 20 – 24 KHz channel

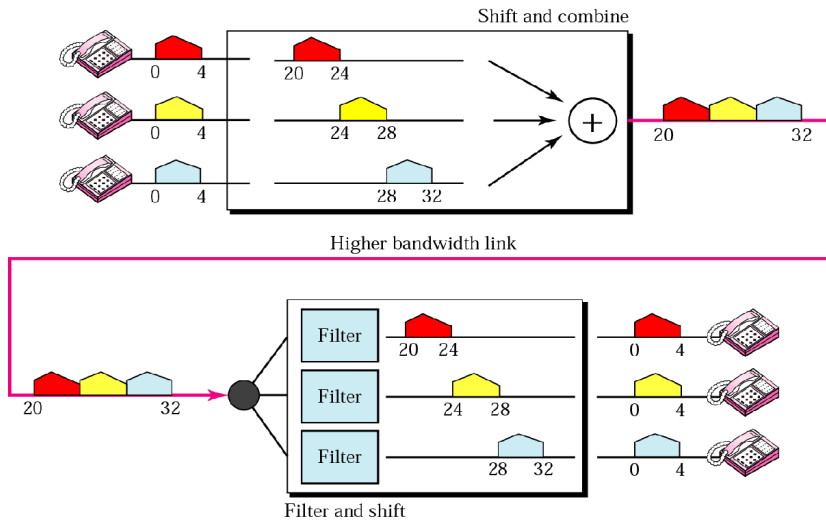
- Use 20.5 KHz to 23.5 KHz, with 0.5 KHz of guard band on each end.

Signal 2: 24 – 28 KHz

Signal 3: 28 – 32 KHz

At receiver, filters are used to isolate each channel, and then the frequency is transformed back to its original range.

FDM



FDM applications

High capacity phone lines

AM radio: 530 KHz to 1700 KHz, 10 KHz bandwidth per station

FM radio: 88 MHz to 108 MHz, 200 KHz bandwidth per station

TV broadcasts: 6 MHz bandwidth per TV channel

First generation cell phones: each user got two 30 KHz channels (sending, receiving).

Wave Division Multiplexing

Essentially same as FDM, except signals are optical and prisms are used to combine/split signals instead of electrical components.

Used to combine signals of different frequencies (i.e. colours) onto one fibre-optic cable

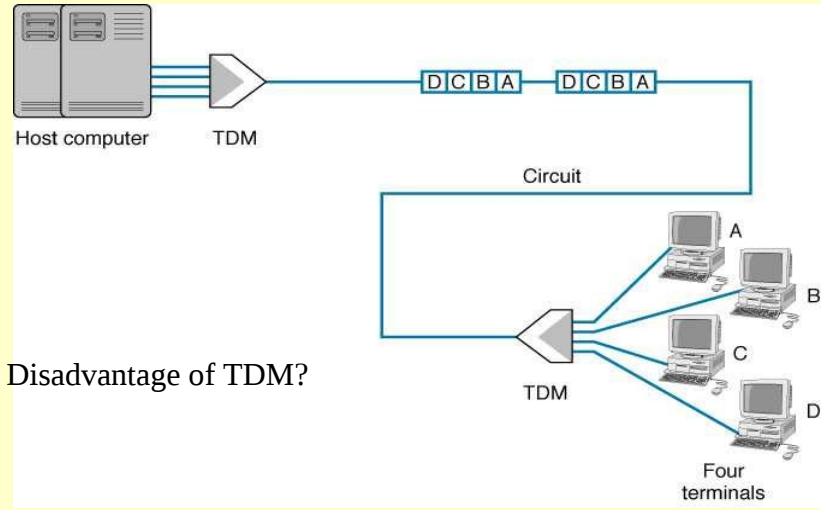
Time Division Multiplexing (TDM)

TDM allows multiple channels by allowing channels to send data by “taking turns”

TDM is sometimes referred to as dividing circuit “vertically”

- [Next slide](#) shows 4 terminals sharing a circuit, with each terminal sending one character at a time
- With TDM, time on circuit is shared equally each channel gets a time slot, whether or not it has any data to send
- TDM is more efficient than FDM, since TDM doesn't use guardbands, so entire capacity can be divided between channels

Time Division Multiplexing



TDM applications

Digital Service lines: DS-n

Implemented as telephone lines: T-n

Service	Phone line	Data rate	# of voice channels
(DS-0)	standard phone line	64 Kb/s	1
DS-1	T-1	1.544 Mb/s	24
DS-2	T-2	6.312 Mb/s	96
DS-3	T-3	44.736 Mb/s	672
DS-4	T-4	274.176 Mb/s	4032

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Code Division Multiplexing (CDM)

CDM used in parts of cellular telephone system and for some satellite communication

CDM relies on an interesting mathematical idea values from **orthogonal vector** spaces can be combined and separated without interference

Each **sender** is assigned a unique binary code C_i that is known as a **chip sequence**

chip sequences are selected to be orthogonal vectors

- Means **dot product** of any two chip sequences is zero

Code Division Multiplexing (CDM)

At any point in time, each sender has value to transmit, V_i

The senders each multiply $C_i \times V_i$ and transmit the results

The senders transmit at the same time and the values are added together

To extract value V_i , a receiver multiplies the sum by C_i

Example

Make example easy to understand, use chip sequence that's only **two bits** long and data values that are **four bits** long

think of the chip sequence as a vector

Next slide lists the values

Code Division Multiplexing

Sender	Chip Sequence	Data Value
A	1 0	1 0 1 0
B	1 1	0 1 1 0

Figure 11.15 Example values for use with code division multiplexing.

Code Division Multiplexing

The first step consists of converting the binary values into vectors that use **-1** to represent **0**:

$$C_1 = (1, -1) \quad V_1 = (1, -1, 1, -1) \quad C_2 = (1, 1) \quad V_2 = (-1, 1, 1, -1)$$

Multiplying $C_1 \times V_1$ and $C_2 \times V_2$ produces:

$$((1, -1), (-1, 1), (1, -1), (-1, 1)) \quad ((-1, -1), (1, 1), (1, 1), (-1, -1))$$

If we think of the resulting values as a combined signal to be transmitted at the same time

the resulting signal will be the sum of the two signals

$$\begin{array}{rcccccccc} & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ + & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ \hline & 0 & -2 & 0 & 2 & 2 & 0 & -2 & 0 \end{array}$$

Code Division Multiplexing

A receiver treats the sequence as a vector
computes product of vector and the chip sequence
treats result as a sequence, and converts the result to binary by interpreting positive values as binary **1** and negative values as **0**

Thus, **Receiver 1** computes:

$$\begin{array}{l} C_1 \rightarrow (1, -1) \cdot \underbrace{((0, -2), (0, 2), (2, 0), (-2, 0))}_{\text{Received data}} \\ \text{to get:} \quad \quad \quad ((0+2), (0-2), (2+0), (-2+0)) \end{array}$$

Interpreting the result as a sequence produces: (2 -2 2 -2)

which becomes the binary value: (1 0 1 0)

note that **1010** is the correct value of V_1

Receiver 2 will extract V_2 from the same transmission

Code division multiple access (CDMA) is a channel access method utilized by various radio communication technologies.

